Chapter 38: Light Waves Behaving as Particles

Group Members:

1. A $75W$ lamp emits light of wavelength $650.nm$. (Useful data: $h = 6.63\times10^{-34}\,J\cdot s$)

   a. Calculate the frequency of the emitted light.

   $$f = \frac{c}{\lambda} = \frac{3.00\times10^{8}\,m/s}{650\times10^{-9}\,m} = 4.615\times10^{14}\,Hz = 4.62\times10^{14}\,Hz$$

   b. How much energy does a photon with this frequency carry?

   Using the Planck's hypothesis, the energy of a photon is given by,

   $$E = hf = 6.63\times10^{-34}\,J\cdot s\left(4.615\times10^{14}\,Hz\right) = 3.06\times10^{-19}\,J$$

   c. Assuming all energy consumed by the lamp goes into emitting this light, how many photons per second does the lamp emit?

   In one second, the lamp consumes $75J$ of energy. So, the number of photons emitted by the lamp is,

   $$N = \frac{75J}{3.06\times10^{-19}\,J} = 2.45\times10^{20}$$

2. In a photoelectric experiment, photons of wavelength $550.nm$ are incident on a metal surface of unknown work function. The stopping potential V_o is found to be $0.861V$. Express all energy quantities in eV.

   (Useful data: $m_e = 9.11\times10^{-31}kg$; $1eV = 1.60\times10^{-19}J$; $hc = 1241\,eV\cdot nm$)

   a. What is the energy of the incident photon?
   b. What is the maximum kinetic energy $K_{\max}$ of the ejected electron?
   c. Determine the work function of the metal.
   d. Lastly, calculate the maximum speed of the ejected electron?

a. The energy of the incident photon is given by,

$$E = hf = \frac{hc}{\lambda} = \frac{1242\,eV\cdot nm}{550nm} = 2.256eV = 2.26eV$$

b. The stopping potential directly gives the maximum kinetic energy of the ejected electron,

$$K_{\max} = eV_o = 0.861eV$$

c. Then, the work function of the metal is given by,

$$W_0 = hf - K_{\max} = 2.256eV - 0.8612eV = 1.40eV$$

d. The maximum kinetic energy of the ejected electron is related to the maximum speed of the ejected electron by $K_{\max} = \frac{1}{2}m_e v^2$. So, solving for v, we have, $v = \sqrt{\frac{2K_{\max}}{m_e}}$.

In Joule, $K_{\max} = 0.8612eV\left(\frac{1.60\times10^{-19}J}{1eV}\right) = 1.378\times10^{-19}J$

And, $v = \sqrt{\frac{2K_{\max}}{m_e}} = \sqrt{\frac{2\left(1.378\times10^{-19}J\right)}{9.11\times10^{-31}kg}} = 5.50\times10^{5}\ m/s$

The speed v of the ejected electron is sufficiently smaller than the speed of light $(v \ll c)$ so it is justified to use the nonrelativistic form of the KE, i.e., $KE = \frac{1}{2}mv^2$.

3. In a Compton Scattering experiment, X-rays of wavelength $\lambda = 0.150\,nm$ are incident on a free electron initially at rest. At a particular observed angle ϕ, the scattered X-rays has a shifted wavelength λ' of $0.154\,nm$.

   (Useful data: $m_e = 9.11\times10^{-31}kg;\ h = 6.63\times10^{-34}J\cdot s;\ hc = 1241eV\cdot nm; 1eV = 1.60\times10^{19}J$)

   a. Calculate the angle ϕ through which the photon is scattered?

      The Compton Shift equation is given by,

      $$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1-\cos\phi)$$

      So,

      $$\phi = \cos^{-1}\left(1 - \frac{m_e c(\lambda' - \lambda)}{h}\right)$$

      $$= \cos^{-1}\left(1 - \frac{(9.11\times10^{-31}kg)(3\times10^{8}\,m/s)(0.004\times10^{-9}\,nm)}{6.63\times10^{-34}J\cdot s}\right)$$

      $$= \cos^{-1}(-0.64887) = 130^\circ$$

   b. What is the energy of the scattered X-rays photon in *eV*?

      $$E = hf = \frac{hc}{\lambda'} = \frac{1241eV\cdot nm}{0.154nm} = 8.058keV = 8.06keV$$

   c. The electron will also be scattered in this process. Use conservation of energy to calculate the kinetic energy of the scattered electron in *eV*.

      By the conservation of energy, the kinetic energy gained by the electron (initially at rest) must be equal to the energy loss by the scattered X-rays photon with respect to the incident X-rays photon, i.e.,

      $$KE_e = hf - hf' = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = \frac{1241eV\cdot nm}{0.150nm} - \frac{1241eV\cdot nm}{0.154nm} = 215eV$$